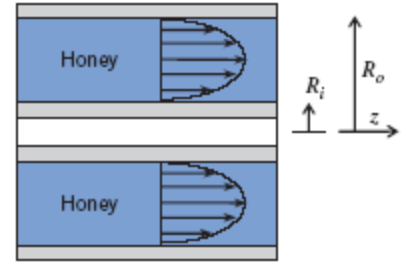


## Problem 2.54

[Difficulty: 3]

**2.54** In a food-processing plant, honey is pumped through an annular tube. The tube is  $L = 2$  m long, with inner and outer radii of  $R_i = 5$  mm and  $R_o = 25$  mm, respectively. The applied pressure difference is  $\Delta p = 125$  kPa, and the honey viscosity is  $\mu = 5 \text{ N} \cdot \text{s}/\text{m}^2$ . The theoretical velocity profile for laminar flow through an annulus is:

$$u_z(r) = \frac{1}{4\mu} \left( \frac{\Delta p}{L} \right) \left[ R_o^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{r}{R_i}\right) \right]$$



Show that the no-slip condition is satisfied by this expression. Find the location at which the shear stress is zero. Find the viscous forces acting on the inner and outer surfaces, and compare these to the force  $\Delta p \pi (R_o^2 - R_i^2)$ . Explain.

**Given:** Data on annular tube

**Find:** Whether no-slip is satisfied; location of zeroshear stress; viscous forces

**Solution:**

The velocity profile is

$$u_z(r) = \frac{1}{4\mu} \cdot \frac{\Delta p}{L} \cdot \left( R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{r}{R_i}\right) \right)$$

Check the no-slip condition. When  $r = R_o$

$$u_z(R_o) = \frac{1}{4\mu} \cdot \frac{\Delta p}{L} \cdot \left( R_i^2 - R_o^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{R_o}{R_i}\right) \right)$$

$$u_z(R_o) = \frac{1}{4\mu} \cdot \frac{\Delta p}{L} \cdot \left[ R_i^2 - R_o^2 + (R_o^2 - R_i^2) \right] = 0$$

When  $r = R_i$

$$u_z(R_i) = \frac{1}{4\mu} \cdot \frac{\Delta p}{L} \cdot \left( R_i^2 - R_i^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{R_i}{R_i}\right) \right) = 0$$

The no-slip condition is satisfied.

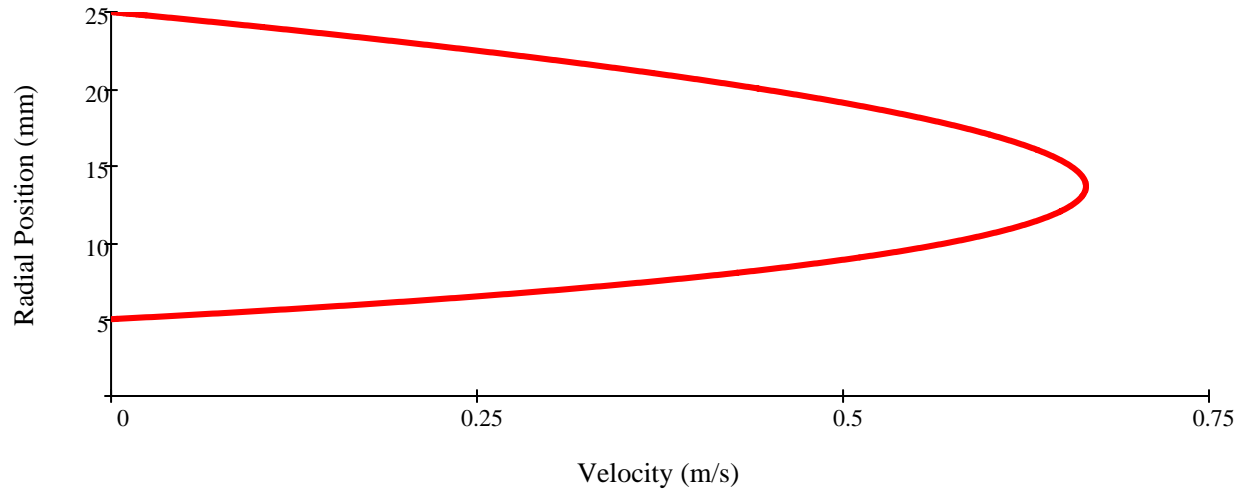
The given data is

$$R_i = 5 \text{ mm} \quad R_o = 25 \text{ mm} \quad \Delta p = 125 \text{ kPa} \quad L = 2 \text{ m}$$

The viscosity of the honey is

$$\mu = 5 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

The plot looks like



For each, shear stress is given by

$$\tau_{rx} = \mu \cdot \frac{du}{dr}$$

$$\tau_{rx} = \mu \cdot \frac{du_z(r)}{dr} = \mu \cdot \frac{d}{dr} \left[ \frac{1}{4 \cdot \mu} \cdot \frac{\Delta p}{L} \cdot \left( R_i^2 - r^2 - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \ln\left(\frac{r}{R_i}\right) \right) \right]$$

Hence

$$\tau_{rx} = \frac{1}{4} \cdot \frac{\Delta p}{L} \cdot \left( -2 \cdot r - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \frac{1}{r} \right)$$

For zero stress

$$-2 \cdot r - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \frac{1}{r} = 0 \quad \text{or} \quad r = \sqrt{\frac{R_i^2 - R_o^2}{2 \cdot \ln\left(\frac{R_i}{R_o}\right)}} \quad r = 13.7 \cdot \text{mm}$$

On the outer surface

$$F_o = \tau_{rx} \cdot A = \frac{1}{4} \cdot \frac{\Delta p}{L} \cdot \left( -2 \cdot R_o - \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)} \cdot \frac{1}{R_o} \right) \cdot 2 \cdot \pi \cdot R_o \cdot L$$

$$F_o = \Delta p \cdot \pi \cdot \left( -R_o^2 - \frac{R_o^2 - R_i^2}{2 \cdot \ln\left(\frac{R_i}{R_o}\right)} \right)$$

$$F_O = 125 \times 10^3 \cdot \frac{\text{N}}{\text{m}^2} \times \pi \times \left[ - \left( 25 \cdot \text{mm} \times \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \right)^2 - \frac{\left[ (25 \cdot \text{mm})^2 - (5 \cdot \text{mm})^2 \right] \times \left( \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \right)}{2 \cdot \ln \left( \frac{5}{25} \right)} \right]$$

$$F_O = -172 \text{ N}$$

On the inner surface

$$F_i = \tau_{rx} \cdot A = \frac{1}{4} \cdot \frac{\Delta p}{L} \cdot \left( -2 \cdot R_i - \frac{R_o^2 - R_i^2}{\ln \left( \frac{R_i}{R_o} \right) \cdot R_i} \right) \cdot 2 \cdot \pi \cdot R_i \cdot L$$

$$F_i = \Delta p \cdot \pi \cdot \left( -R_i^2 - \frac{R_o^2 - R_i^2}{2 \cdot \ln \left( \frac{R_i}{R_o} \right)} \right)$$

Hence

$$F_i = 125 \times 10^3 \cdot \frac{\text{N}}{\text{m}^2} \times \pi \times \left[ - \left( 5 \cdot \text{mm} \times \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \right)^2 - \frac{\left[ (25 \cdot \text{mm})^2 - (5 \cdot \text{mm})^2 \right] \times \left( \frac{1 \cdot \text{m}}{1000 \cdot \text{mm}} \right)}{2 \cdot \ln \left( \frac{5}{25} \right)} \right]$$

$$F_i = 63.4 \text{ N}$$

Note that

$$F_O - F_i = -236 \text{ N} \quad \text{and} \quad \Delta p \cdot \pi \cdot (R_o^2 - R_i^2) = 236 \text{ N}$$

The net pressure force just balances the net viscous force!